B.Sc Semester - IV

Paper - Mat C-8

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Model Questions - Analysis II

Group A

(1) Answer the following questions. Each question carries 2 marks.

(a) Discuss about the Comparison Test.

(b) State Abel's test

(c) Define Dirichlet's test.

(d) Define Gamma & Beta Function.

(e) Define double integral.

(f) State Dirichlet's Theorem for three variables.

(g) Change the order of integration in the integral $\int_0^a \int_0^x f(x,y)\,dx\,dy$.

(h) Transform $\iint v \, dx\, dy$ into Polar Co-ordinate.

(i) Show that $\int_0^\infty e^{-kx} x^{n-1} dx = \frac{\Gamma(n)}{k^n}$, $n>0$, $k>0$

(j) Define improper integral.

(k) Write duplication formula.

(l) State Green's theorem.

(m) Write the statement of Stokes Theorem

(n) Write the statement of Gauss Divergence Theorem.

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(11) Prove that $\iint_S \vec{r}\cdot\vec{n}\,ds = 3V$, where the surface S encloses the volume V.

Group A

Answer the following questions. Each question carries 8 marks.

1) Test the convergence of the integral $\int_0^\infty \frac{\cos x}{1+x^2}\,dx$.

2) Test the convergence of $\int_a^\infty \left(1-e^{-x}\right)\frac{\cos x}{x^2}\,dx$ where $a>0$.

(3) Show that $\int_0^\infty \sin x^2\,dx$ is convergent.

(4) Show that $\int_0^\infty e^{-ax}\cdot\frac{\sin x}{x}\,dx$ is convergent when $\alpha \geq 0$.

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(5) prove that the Gamma function

$$\Gamma(n) = \int_0^\infty x^{n-1} e^{-x}\, dx, \text{ where } n>0 \text{ is}$$

Convergent.

(6) Discuss the Convergence of the Beta function $\int_0^1 x^{m-1}(1-x)^{n-1}dx$.

(7) Establish the relation

$$B(l,m) = \frac{\Gamma(l)\,\Gamma(m)}{\Gamma(l+m)}$$

(8) Prove that

$$\Gamma(n)\,\Gamma\left(n+\frac{1}{2}\right) = \frac{\sqrt{\pi}}{2^{2n-1}}\,\Gamma(2n)$$

where n is an integer.

Group B

(9) State Liouville's Extension of Dirichlet's theorem and prove it.

(10) Evaluate $\iiint x^{-1/2} y^{-1/2} z^{-1/2} (1-x-y-z)^{1/2}\, dx\, dy\, dz$

extended to all positive values of the variables subject to the condition $x+y+z<1$.

(11) Change the order of integration in $\int_0^a \int_0^{\sqrt{a^2-x^2}} f(x,y)\,dx\,dy$.

Group C (Vector Integration)

(13) If $\vec{F} = 3xy\,\vec{i} - y^2\vec{j}$, Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where C is the curve in the xy plane, $y = 2x^2$, from $(0,0)$ to $(1,2)$.

(14) Evaluate $\int_C \vec{F}\cdot d\vec{r}$ where $\vec{F} = (x^2+y^2)\vec{i} - 2xy\,\vec{j}$, where C is the rectangle in the xy plane bounded by $y=0$, $x=a$, $y=b$, $x=0$.

(15) Evaluate $\iint \vec{F}\cdot\vec{n}\,ds$ where $F = yz\vec{i} + zx\vec{j} + xy\vec{k}$ and S is that part of the surface of the sphere $x^2+y^2+z^2=1$ which lies in the first octant.

(16) Verify Green's theorem in the plane for $\oint_C (xy+y^2)\,dx + x^2\,dy$ where C is the closed curve of the region bounded by $y=x$ & $y=x^2$.

(17) Verify divergence theorem for

$$F = (x^2-yz)\vec{i} + (y^2-zx)\vec{j} + (z^2-xy)\vec{k}$$

taken over the rectangular parallelepiped $0 \le x \le a$, $0 \le y \le b$, $0 \le z \le c$.

(18) State and prove Stoke's theorem.

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(19) Verify Stoke's theorem for

$$F = (x^2+y^2)\vec{i} - 2xy\vec{j}$$

taken round the rectangle bounded by

$$x = \pm a, \quad y = 0, \quad y = b.$$

(20) State and Prove Gauss - Divergence Theorem.